

Automatic Guidance Concept for VTOL Aircraft

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A simplified mathematical model of a VTOL aircraft is developed that treats ground range-to-go as the independent variable, altitude and crossrange as state variables, and the three commanded velocity components as control variables. By considering perturbations about a nominal path and minimizing a performance index which is a quadratic function of the state and control variable deviations, a simple linear guidance law is obtained. To implement the guidance scheme, the feedback gains would be precalculated and stored, along with the nominal state and control variable histories, as functions of range-to-go in the onboard computer. During flight, the actual state variables would be measured and their deviations from nominal used to calculate optimal corrections to the stored nominal control variables. Simulation results with the tandem-rotor CH-46C helicopter demonstrate the guidance scheme's ability to accommodate a variety of off-nominal conditions.

Nomenclature

A	= penalty matrix on enroute state variable deviations; defined in Eq. (15)
B	= penalty matrix on enroute control deviations; defined in Eq. (16)
C	= matrix of guidance feedback gains; defined in Eq. (18)
$C_{V_x, y} \dots$	= feedback gains for V_x due to errors in y , etc.; 1/sec
F	= homogeneous response matrix of linearized, simplified model; defined in Eq. (11)
F_l, F_m, F_n	= aerodynamic rolling, pitching and yawing moments; N - m
F_x, F_y, F_z	= aerodynamic axial, lateral and normal forces; N
G	= forced response matrix of linearized, simplified model; defined in Eq. (12)
J	= performance index for quadratic synthesis; defined in Eq. (13)
$K(\cdot)$	= flight control system gains; see Fig. 5
$L(\cdot)$	= $\partial F_l / \partial(\cdot)$ = trim rolling moment stability derivatives
$M(\cdot)$	= $\partial F_m / \partial(\cdot)$ = trim pitching moment stability derivatives
$N(\cdot)$	= $\partial F_n / \partial(\cdot)$ = trim yawing moment stability derivatives
R	= ground range to touchdown; defined in Eq. (4); m
S	= Riccati matrix; defined by Eqs. (19) and (20)
V	= $(V_x^2 + V_y^2 + V_z^2)^{1/2}$ = magnitude of inertial velocity; see Fig. 2; m/sec
V_H, V_V	= horizontal and vertical components of relative velocity; defined in Eqs. (33) and (34); m/sec
V_x, V_y, V_z	= inertial components of aircraft velocity; see Fig. 2; m/sec
$X(\cdot)$	= $\partial F_x / \partial(\cdot)$ = axial force stability derivatives

$Y(\cdot)$	= $\partial F_y / \partial(\cdot)$ = lateral force stability derivatives
$Z(\cdot)$	= $\partial F_z / \partial(\cdot)$ = normal force stability derivatives
g	= acceleration of gravity; 9.80 m/sec ²
h	= $-z$ = altitude above touchdown; m
m	= mass of aircraft; kg
p, q, r	= roll, pitch and yaw rates; rad/sec
s	= Laplace operator; 1/sec
t	= time; sec
u	= control vector perturbation for simplified model; defined in Eq. (10)
u, v, w	= components of relative velocity in body axes; m/sec
w_x, w_y, w_z	= inertial components of wind velocity, m/sec
x	= state vector perturbation for simplified model; defined in Eq. (10)
x, y, z	= downrange, crossrange and vertical position coordinates; see Fig. 2; m
$\Delta(\cdot)$	= $(\cdot) - (\cdot)_o$ = deviation of indicated quantity from its trim value
β	= $\tan^{-1}(v/u)$ = sideslip angle of fuselage
γ	= $\sin^{-1}(-V_z/V)$ = flight path angle; see Fig. 2
$\delta(\cdot)$	= $(\cdot) - (\cdot)_N$ = deviation of indicated quantity from its nominal value
δ_a	= roll cyclic control deflection; cm
δ_c	= gang collective control deflection; cm
δ_e	= differential collective or longitudinal cyclic control deflection; cm
δ_r	= yaw cyclic control deflection; cm
θ, ϕ, ψ	= pitch, roll and yaw attitudes of fuselage
λ	= $\tan^{-1}(V_y/V_x)$ = heading angle; see Fig. 2
$\tau(\cdot)$	= flight control system lag time constants; see Fig. 5

Subscripts

E	= error between commanded and existing values
G	= difference to be gained
I	= integral of subscripted quantity
N	= nominal value
c	= commanded value
f	= final value
i	= initial value
m	= maximum desirable value
o	= trim value

Miscellaneous

(\cdot)	= derivative of indicated quantity
$(\cdot)^T$	= transpose of indicated matrix
$(\cdot)^{-1}$	= inverse of indicated matrix

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I. Introduction

IN the past few years, the possible commercial use of vertical/short takeoff and landing (V/STOL) aircraft to help relieve the air traffic problem has received wide attention. Responding to this interest, the NASA Electronics Research Center undertook a program to "develop and demonstrate the avionics system technologies that would make V/STOL aircraft all-weather operations both technically and economically feasible on civil air route structures and in air traffic environments representative of the 1975 to 1985 time period."¹

During phase 1 of the program, the feasibility of using an inertial guidance system/nav-aid system for terminal guidance and navigation of V/STOL aircraft was evaluated by means of a Gemini inertial platform and computer onboard an H-19 helicopter and a CH-46C helicopter. Flights were made both with and without position updates from a ground-based radar, and test results were used to determine update requirements as a function of sensor performance. The over-all objective of phase 2 of the program was to evaluate and demonstrate the technical and commercial feasibility of using integrated, digital strapdown/nav-aid system concepts for flight control, guidance and navigation of V/STOL aircraft. This paper describes a study of the guidance subsystem that has been conducted during the phase 2 effort. The research objective was to develop and evaluate automatic guidance modes and associated software for V/STOL aircraft.

II. Analytical Development

Figure 1 is a block diagram of the automatic V/STOL avionics system showing the flow of information and the relationships of the three principal subsystems (navigation, guidance, and flight control). The function of the guidance system is to control the position and velocity of the aircraft in order to achieve specified flight objectives, e.g., to transport passengers from one airport to another under zero visibility conditions. To perform this task, the guidance system requires position and velocity information from the navigation system. It uses these data to generate velocity commands for the flight control system, which provides the direct control of the vehicle maneuvers by displacement of the appropriate aircraft controls.

Simplified Model

The dynamics of the flight control system and the aircraft introduce delays in the response to velocity commands. However, the response time of the controlled vehicle (on the order of a few seconds), is short relative to time periods of interest to the guidance system (on the order of tens of seconds). Hence, from the viewpoint of the guidance system, velocity commands are achieved with negligible time delay.

The guidance system proposed here was developed with the simplest possible representation of the controlled aircraft—a perfect autopilot. That is, the aircraft's true velocity components were assumed to be identical to their respective commanded values

$$\dot{x} = V_x = V_{x_c} \quad (1)$$

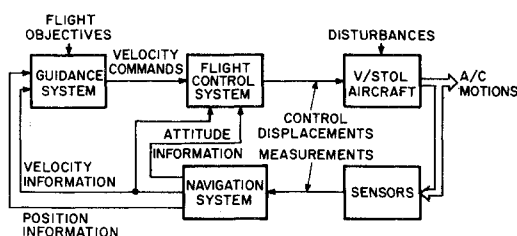


Fig. 1 Block diagram of the V/STOL avionics system.

$$\dot{y} = V_y = V_{y_c} \quad (2)$$

$$\dot{z} = V_z = V_{z_c} \quad (3)$$

where x , y and z are the position coordinates of the aircraft in the inertial coordinate system illustrated in Fig. 2. The origin of the coordinate system is at the intended runway touchdown point; the x -axis is in the horizontal plane, parallel to the runway and positive in the landing direction; the z -axis is positive down along the local vertical; and the y -axis forms a right-hand orthogonal system. V_x , V_y and V_z are the three components of inertial velocity; V_{x_c} , V_{y_c} and V_{z_c} are the corresponding velocity commands. Equations (1-3) describe a simple dynamic system having three state variables (x, y, z) and three control variables ($V_{x_c}, V_{y_c}, V_{z_c}$), with time (t) being the independent variable.

The vehicle model can be reduced even further, since time is not of great importance, by using a monotonic function of the position coordinates as the independent variable. For most flights the ground range-to-go R is convenient for this purpose

$$R = (x^2 + y^2)^{1/2} \quad (4)$$

Differentiating Eq. (4) with respect to time and using Eqs. (1) and (2) gives

$$\dot{R} = (xV_{x_c} + yV_{y_c})/R \quad (5)$$

Dividing Eqs. (2) and (3) by Eq. (5), we obtain

$$dy/dR = RV_{y_c}/(xV_{x_c} + yV_{y_c}) \quad (6)$$

$$dz/dR = RV_{z_c}/(xV_{x_c} + yV_{y_c}) \quad (7)$$

where x is determined from

$$x = -(R^2 - y^2)^{1/2} \quad (8)$$

Eqs. (6-8) form the approximate model of the controlled aircraft used for synthesis of the guidance system. The two state variables are y and z ; the three control variables are V_{x_c} , V_{y_c} and V_{z_c} ; R is the independent variable; and the third position coordinate x is a dependent parameter given by Eq. (8).

Perturbation Equations

The simplified equations of motion, Eqs. (6) and (7), may be linearized by taking first-order perturbations of the equations about a nominal trajectory

$$d\mathbf{x}/dR = \mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{u} \quad (9)$$

where the state vector perturbation \mathbf{x} and the control vector perturbation \mathbf{u} are defined by

$$\mathbf{x} = \begin{bmatrix} \delta y \\ \delta z \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \delta V_{x_c} \\ \delta V_{y_c} \\ \delta V_{z_c} \end{bmatrix} \quad (10)$$

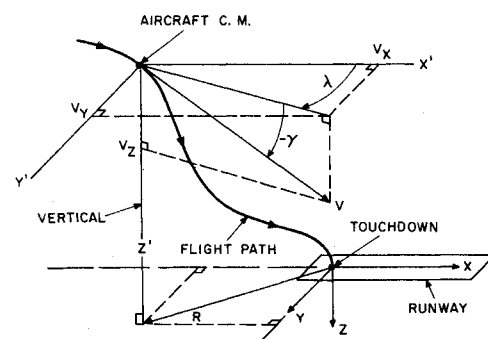


Fig. 2 Coordinate system and nomenclature for guidance analysis.

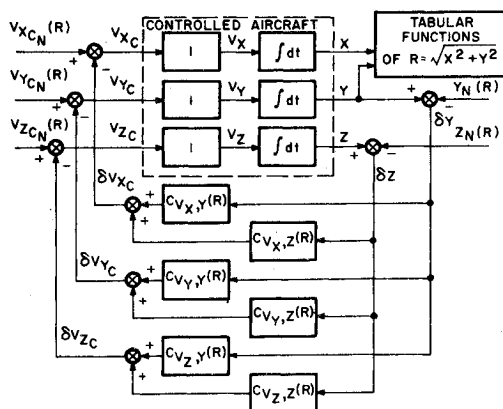


Fig. 3 Velocity-command guidance system with simple model of controlled aircraft.

and the matrices \mathbf{F} and \mathbf{G} are given by

$$\mathbf{F} = \frac{R(y_N V_{x_{cN}} - x_N V_{y_{cN}})}{x_N(x_N V_{x_{cN}} + y_N V_{y_{cN}})^2} \begin{bmatrix} V_{y_{cN}} & 0 \\ V_{x_{cN}} & 0 \end{bmatrix} \quad (11)$$

$$\mathbf{G} = \frac{R}{(x_N V_{x_{cN}} + y_N V_{y_{cN}})^2} \begin{bmatrix} -x_N V_{y_{cN}} x_N V_{x_{cN}}, & 0 \\ -x_N V_{z_{cN}}, -y_N V_{z_{cN}}, x_N V_{x_{cN}} + y_N V_{y_{cN}} \end{bmatrix} \quad (12)$$

In the above equations, the perturbation quantities are defined by $\delta(\cdot) = (\cdot) - (\cdot)_N$, and the subscript N denotes the nominal value of the indicated quantity.

Quadratic Synthesis

To obtain a linear feedback guidance law, it is convenient to choose a performance index that is a quadratic function of the control and state variable perturbations (see, for example, Ref. 2)

$$J = \frac{1}{2} (\mathbf{x}^T \mathbf{S}_f \mathbf{x})_{R=R_f} + \frac{1}{2} \int_{R_i}^{R_f} [\mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{u}^T \mathbf{B} \mathbf{u}] dR \quad (13)$$

where R_i and R_f are, respectively, the initial and final range. The matrices \mathbf{S}_f and \mathbf{A} weight the penalties associated with terminal and enroute state variable errors, and the matrix \mathbf{B} penalizes control deviations during the flight

$$\mathbf{S}_f = \begin{bmatrix} \delta y_f^{-2} & 0 \\ 0 & \delta z_f^{-2} \end{bmatrix} \quad (14)$$

$$\mathbf{A} = \frac{1}{R_f - R_i} \begin{bmatrix} \delta y_m^{-2} & 0 \\ 0 & \delta z_m^{-2} \end{bmatrix} \quad (15)$$

$$\mathbf{B} = \frac{1}{R_f - R_i} \begin{bmatrix} \delta V_{x_{cm}}^{-2} & 0 & 0 \\ 0 & \delta V_{y_{cm}}^{-2} & 0 \\ 0 & 0 & \delta V_{z_{cm}}^{-2} \end{bmatrix} \quad (16)$$

The parameters $\delta y_f, \delta z_f, \delta y_m, \delta z_m, \delta V_{x_{cm}}, \delta V_{y_{cm}}, \delta V_{z_{cm}}$ must be chosen to provide satisfactory enroute and terminal guidance within acceptable limitations on $\delta V_{x_c}, \delta V_{y_c}$ and δV_{z_c} .

The minimization of the performance criterion, Eq. (13), subject to the perturbation equations of motion, Eq. (9), yields the linear feedback guidance law

$$\mathbf{u} = -\mathbf{C}(R)\mathbf{x} \quad (17)$$

The feedback gain matrix $\mathbf{C}(R)$ is defined by

$$\mathbf{C} = \mathbf{B}^{-1}\mathbf{G}^T\mathbf{S} \quad (18)$$

where \mathbf{S} is the solution of the matrix Riccati equation

$$d\mathbf{S}/dR = -\mathbf{S}\mathbf{F} - \mathbf{F}^T\mathbf{S} + \mathbf{S}\mathbf{G}\mathbf{B}^{-1}\mathbf{G}^T\mathbf{S} - \mathbf{A} \quad (19)$$

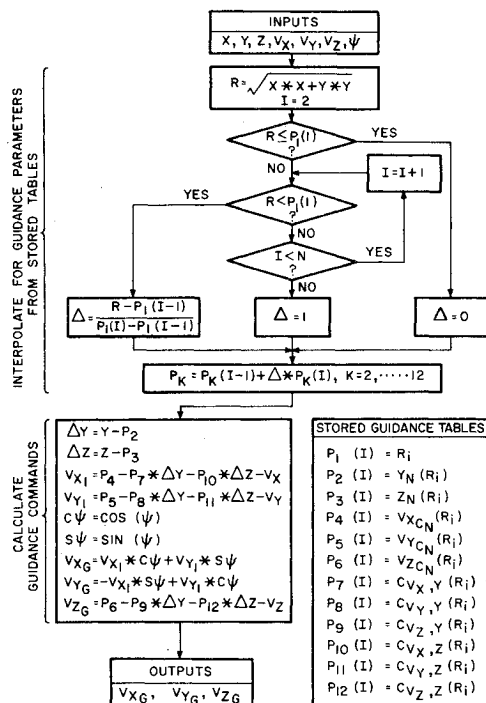


Fig. 4 Computer flow diagram for guidance scheme.

with the boundary condition

$$\mathbf{S}(R_f) = \mathbf{S}_f \quad (20)$$

III. Implementation of the Guidance Concept

In expanded form the guidance law, Eq. (17), is

$$V_{x_c} = V_{x_{cN}} - C_{V_{x,u}}(y - y_N) - C_{V_{x,z}}(z - z_N) \quad (21)$$

$$V_{y_c} = V_{y_{cN}} - C_{V_{y,u}}(y - y_N) - C_{V_{y,z}}(z - z_N) \quad (22)$$

$$V_{z_c} = V_{z_{cN}} - C_{V_{z,u}}(y - y_N) - C_{V_{z,z}}(z - z_N) \quad (23)$$

Figure 3 shows a block diagram of the velocity-command guidance system with the simple model of the controlled aircraft used in the guidance synthesis. In the simulation described subsequently, the dashed block "Controlled Aircraft" is replaced by the flight control system and a rigid body dynamic model of the aircraft.

To comply with the guidance/flight control interface, the guidance system velocity commands were required as the difference between the desired and actual velocity components in a "Vertical Heading" reference frame. This frame, which was selected for convenience in a manually operated mode, is defined as follows: the origin is at the aircraft center of mass; the x -axis is the projection of the body-fixed x -axis onto the local horizontal plane; the z -axis is vertical and directed downward; and the y -axis completes the orthogonal triad. Thus, the transformation from the inertial frame to the "Vertical Heading" frame involves a rotation through the yaw angle ψ about the z -axis. As a result, the guidance system outputs are given by

$$V_{xG} = (V_{x_c} - V_x) \cos \psi + (V_{y_c} - V_y) \sin \psi \quad (24)$$

$$V_{yG} = -(V_{x_c} - V_x) \sin \psi + (V_{y_c} - V_y) \cos \psi \quad (25)$$

$$V_{zG} = V_{z_c} - V_z \quad (26)$$

Figure 4 is a flow diagram for the implementation of the guidance algorithm, Eqs. (21-26). The inputs required are the three inertial position coordinates (x, y, z), the three inertial velocity components (V_x, V_y, V_z), and the heading angle (ψ); the outputs are the three components of velocity-to-be-gained in the "Vertical Heading" frame (V_{xG}, V_{yG}, V_{zG}).

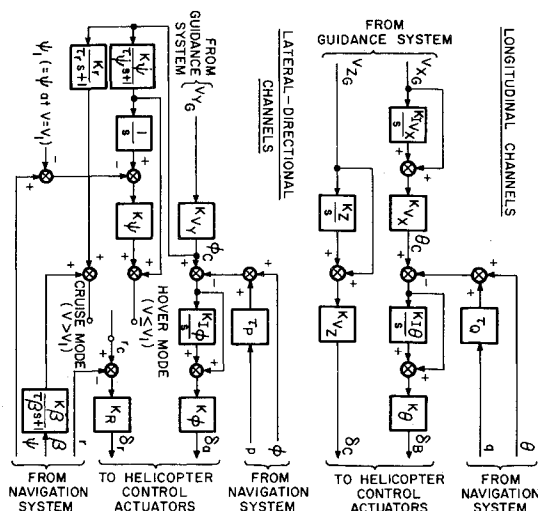


Fig. 5 Velocity-command flight control system for CH-46C helicopter.

Fixed data storage is required for twelve guidance quantities: the independent variable (R); the five nominal trajectory histories ($y_N, z_N, V_{xN}, V_{yN}, V_{zN}$); and the six feedback gains ($C_{V_x, y}, C_{V_y, y}, C_{V_z, y}, C_{V_x, z}, C_{V_y, z}, C_{V_z, z}$). From the results of the simulations conducted with the guidance scheme, it is estimated that approximately 25 sets of data points should be sufficient to represent most nominal trajectories.

Table 1 provides a summary of the estimated computer storage and calculation time requirements for the guidance scheme. The data in Table 1 are based on the flow chart of Fig. 4 and the simulation experience gathered during this investigation. It is apparent that the storage requirements for a single nominal trajectory are very modest, so that a number of different nominals might be carried along for greater flexibility and for contingencies. In fact, six nominals would require only about 2,000 words of memory for the entire guidance function. The maximum time required for the guidance calculations is less than 2 msec; hence a sample rate of 8–10/sec would consume less than 5% of the computer's time.[†]

Although inflight switching from one nominal to another was not investigated during the study, this should not present any major difficulties. If the new nominal were substantially different at the switching point, it is likely that large velocity vector changes would be commanded too suddenly for passenger comfort. However, limits placed on the velocity-to-be-gained and lags in the dynamic response of the flight control system and of the aircraft itself would signifi-

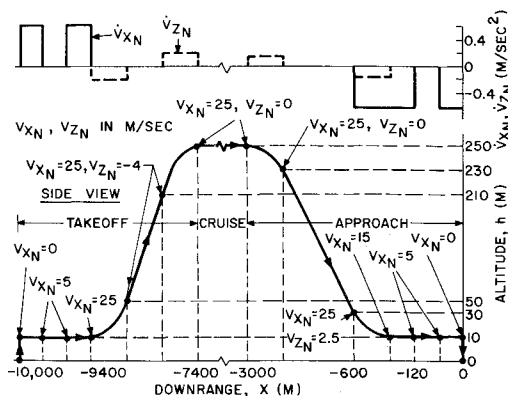


Fig. 6 Nominal VTOL planar trajectory.

[†] Times based on operation speeds for IBM 4PI/CP-2 Computer.

Table 1 Estimated computer requirements

Storage (Words)		Speed (μ sec) ^a		
		Operation	No.	Time
Permanent		\pm @ 3.8	204	775
Guidance tables (1 nominal)	300	\times @ 11.5	23	265
Program	120	\div @ 46.3	1	46
Temporary	30	$\sqrt{}$ @ 250	1	250
Margin ($\approx 10\%$)	50	\sin @ 244	1	244
Total	500	\cos @ 230	1	230
		store @ 2.5	55	135
		margin ($\approx 10\%$)		195
Total				2140

^a Times based on operation speeds for IBM 4PI/CP-2 Computer.

cantly attenuate the resulting accelerations. If the response was not entirely satisfactory, an alternate approach would be to carry a set of simple transition paths to connect the old and new nominals. These transition nominals might, for example, consist of constant climb and descent rates for vertical matching, and connected segments of straight lines and constant radius turns in the horizontal plane.³

IV. Digital Simulation

Aerodynamic Model

The performance of the guidance scheme was evaluated by means of a six-degree-of-freedom digital computer simulation of the tandem-rotor CH-46C helicopter.^{**} A linearized "near-trim" dynamic model of the aircraft was used to calculate the aerodynamic forces and moments as functions of altitude, velocity, the three vehicle attitudes (yaw, pitch, roll) and their rates, and the four control displacements (differential collective, gang collective, roll cyclic, yaw cyclic). In body axes, the forces and moments are

$$F_x = mg \sin \theta_o + X_u \Delta u + X_w \Delta w + X_q q + X_{\delta_e} \Delta \delta_e + X_{\delta_c} \Delta \delta_c \quad (27)$$

$$F_y = Y_v v + Y_p p + Y_r r + Y_{\delta_a} \Delta \delta_a + Y_{\delta_r} \Delta \delta_r \quad (28)$$

$$F_z = -mg \cos \theta_o + Z_u \Delta u + Z_w \Delta w + Z_q q + Z_{\delta_e} \Delta \delta_e + Z_{\delta_c} \Delta \delta_c \quad (29)$$

$$F_l = L_v v + L_p p + L_r r + L_{\delta_a} \Delta \delta_a + L_{\delta_r} \Delta \delta_r \quad (30)$$

$$F_m = M_u \Delta u + M_w \Delta w + M_q q + M_{\delta_e} \Delta \delta_e + M_{\delta_c} \Delta \delta_c \quad (31)$$

$$F_n = N_v v + N_p p + N_r r + N_{\delta_a} \Delta \delta_a + N_{\delta_r} \Delta \delta_r \quad (32)$$

where $\Delta(\) = (\) - (\)_o$ and the subscript o denotes the trim value of the indicated quantity. The trim values $\theta_o, \delta_{e_o}, \delta_{c_o}, \delta_{a_o}, \delta_{r_o}$, and the stability derivatives X_u, X_w , etc., were linearly interpolated from data tables as functions of the horizontal and vertical velocity components relative to the air, V_H and V_V

$$V_H = [(V_x - w_x)^2 + (V_y - w_y)^2]^{1/2} \quad (33)$$

$$V_V = V_z - w_z \quad (34)$$

where w_x, w_y and w_z are the inertial wind components. A complete listing of the simulation program and a tabulation of the aerodynamic and physical data assumed for the aircraft are presented in Ref. 5.

Flight Control System

The velocity-command flight control system used in the simulation was developed for the CH-46C at NASA ERC.

^{**} The simulation program was a modified version of Program LIFT.⁴

Table 2 Weighting parameters used to calculate guidance feedback gains

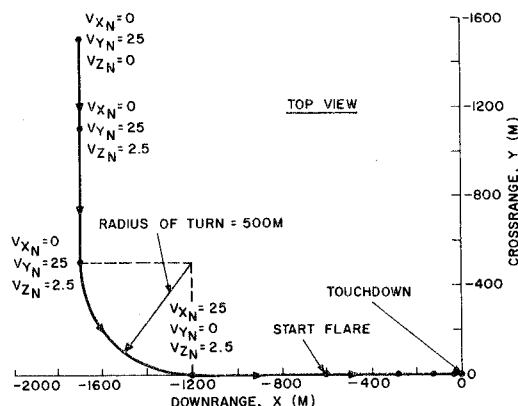
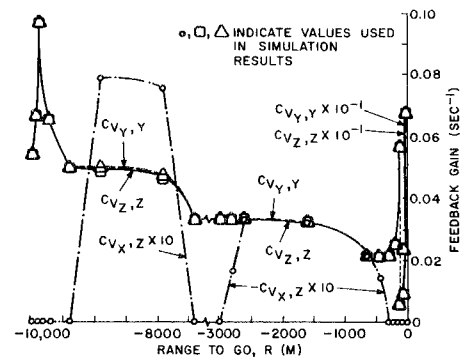
	Takeoff	Cruise	Approaches
δy_m	10.0 m	15.0 m	15.0 m
δz_m	10.0 m	15.0 m	15.0 m
δy_f	27.8 m	∞	5.0 m
δz_f	27.8 m	∞	5.0 m
$\delta V_{x_{cm}}$	0.5 m/sec	0.5 m/sec	0.5 m/sec
$\delta V_{y_{cm}}$	0.5 m/sec	0.5 m/sec	0.5 m/sec
$\delta V_{z_{cm}}$	0.5 m/sec	0.5 m/sec	0.5 m/sec

The longitudinal and lateral-directional channels of this system are shown in Fig. 5. Simulation of the flight control system required eight integrations, three for the longitudinal channels and five for the lateral-directional channels. In Fig. 5, V_1 is the airspeed at which the flight control system switches from a low-speed hover mode for heading control, to a high-speed cruise mode for sideslip control. To prevent large transients when switching from cruise to hover mode, the yaw angle feedback is biased by the value which exists at mode switch, ψ_1 .

Nominal Trajectory

Figure 6 shows the nominal trajectory used to evaluate the proposed guidance scheme. It consists of several piecewise-constant vertical and horizontal acceleration segments, indicated at the top of the figure. A planar flight path having three phases (takeoff, cruise, approach) was selected. The total distance traversed, shown as 10 km in Fig. 6, is easily adjusted by modifying the length of the (steady state) cruise phase. The takeoff phase of the trajectory, which was based on discussions with CH-46C pilots, begins with a vertical liftoff to hover 10 m above the runway. This is followed by a constant horizontal acceleration to 5 m/sec, which is maintained for 20 sec to provide the pilot an opportunity to verify system performance. The forward velocity is then increased to 25 m/sec, at which point the transition to a steady climb rate of 4 m/sec is initiated. This rate of ascent is held until an altitude of 210 m, where the leveling off maneuver begins. The cruise phase commences when the helicopter has attained level flight at an altitude of 250 m, and is maintained until the aircraft is 3000 m from touchdown.

The approach phase of the trajectory is the most crucial in terms of accuracy requirements. A constant vertical acceleration transition is initiated at 3000 m from touchdown and is completed when a flight path angle of -0.1 rad has been achieved. The constant 2.5 m/sec rate of descent is maintained until an altitude of 30 m is reached, at which point the flare maneuver is commenced. The flare itself

**Fig. 7** Nominal 90° approach trajectory.**Fig. 8** Feedback gains for planar nominal trajectory.

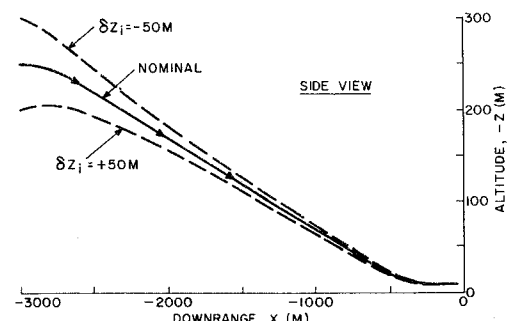
consists of constant vertical and horizontal accelerations such that the aircraft levels off at an altitude of 10 m. The horizontal deceleration is continued until the velocity has been reduced to 5 m/sec. This speed is held for 20 sec to again provide the pilot an opportunity to verify system performance. The remainder of the trajectory consists of a final deceleration to hover above the runway followed by a vertical descent to touchdown.

In addition to the complete nominal path just described, an approach phase nominal was devised to examine the system performance during a 90° turning maneuver. This 90° approach nominal, whose ground track is shown in Fig. 7, was obtained from the planar approach of Fig. 6 by "wrapping" it around a hypothetical vertical cylinder with a radius of 500 m, beginning at $x = -1200$ m. Thus, the two approaches are identical for $R \leq 1200$ m.

Guidance Feedback Gains

Using the nominal trajectories just described, the guidance feedback gains were calculated for each phase by integrating Eq. (19) backwards, i.e., away from the touchdown point. The weighting parameters which were used in these calculations are given in Table 2. Except for the cruise phase, which is a steady-state flight condition, the weighting parameters penalize both enroute and terminal deviations from the nominal trajectory. The values of δy_f and δz_f for the takeoff phase were selected so that the gains at the end of this phase matched the constant cruise gains.

At either end of the nominal trajectory, the aircraft velocity goes to zero. As a result, the rate of change of the independent variable becomes zero, while the nonzero feedback gains become infinite. To avoid these difficulties, the initial and final ranges used in the calculations were 20 m from the actual ends of the trajectory (i.e., $R_i = 9980$ m and $R_f = 20$ m), where in both cases, $V_{x_N} = 5$ /sec. The resulting feedback gains are shown in Fig. 8. For a planar nominal trajectory ($V_{y_N} = y_N = 0$), three of the six gains are always

**Fig. 9** Effects of initial altitude errors on straight-in approach.

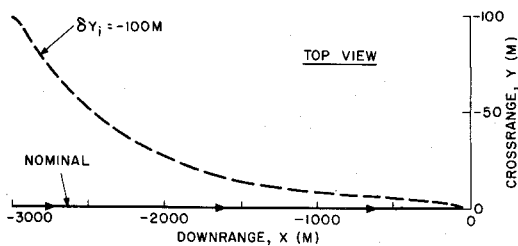


Fig. 10 Effect of initial crossrange error on straight-in approach.

zero: $C_{V_{x,y}}$, $C_{V_{z,y}}$ and $C_{V_{y,z}}$. Furthermore, during level flight ($V_{z,N} = 0$) $C_{V_{x,z}}$ also vanishes. It is also interesting that for these weighting parameters, the remaining two gains, $C_{V_{y,y}}$ and $C_{V_{z,z}}$, are nearly identical throughout the entire flight. Consequently, a savings in computer storage might be realized during a planar flight since only two gain histories are required.

To simulate the guidance system's performance, the nominal trajectory histories and the feedback gains had to be stored as functions of the range to go. The small symbols in Fig. 8 indicate the stored gain values used for the planar nominal.

Simulation Results

A large number of flights were simulated with a variety of off-nominal conditions to evaluate the guidance scheme's performance. The results presented here are representative of all the simulation studies. The approach phase results were selected since this is the most critical portion of the flight in terms of guidance accuracy.

Figure 9 is a side view of the straight-in approach with nominal conditions and with initial altitude errors of ± 50 m. In each case, the proposed guidance scheme smoothly "funnels" the aircraft back toward the nominal path and into the nominal final conditions. Figure 10 compares the projected ground tracks for straight-in approaches with nominal initial conditions and with an initial crossrange error of -100 m. Again, the guidance scheme performs very satisfactorily, steering the helicopter back toward the nominal path and eventually into the nominal terminal point. The effects of an initial heading error of 20° are shown in Figure 11. The guidance scheme corrects the heading before a crossrange deviation of 10 m develops, and then smoothly guides the aircraft back to the nominal path.

The effects of crosswinds during the straight-in approach are illustrated in Fig. 11. Two wind profiles are presented: 1) a constant crosswind of 10 m/sec, and 2) a constant-shear lateral wind, which varies linearly from 10 m/sec at an altitude of 250 m down to 5 m/sec at sea level. The aircraft is initially blown to the right of the nominal path, but compensates for the wind by "crabbing" (i.e. yawing) to the left. The lateral response which begins at $x = -600$ m is caused by the changing trim conditions and aerodynamics during the flare and forward deceleration in conjunction with the crabbed attitude of the aircraft.

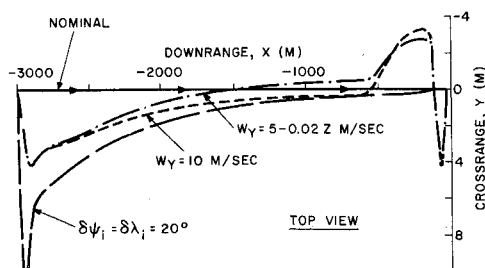


Fig. 11 Effects of initial heading error and crosswinds on straight-in approach.

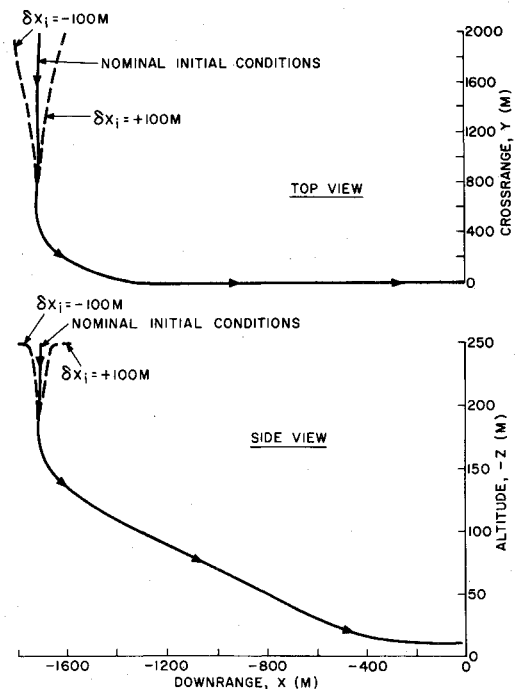


Fig. 12 Effects of initial downrange errors on 90° approach.

Figure 12 presents the side and top views of 90° approaches with nominal initial conditions and with initial downrange errors of ± 100 m. These results, which are typical of all the 90° approach simulations, illustrate that the guidance scheme performs satisfactorily for a turning approach as well as a straight one.

V. Conclusions

The simple feedback guidance scheme described here is capable of automatically and accurately steering a VTOL aircraft during the takeoff, cruise and approach phases of flight. The proposed guidance concept can accommodate large initial condition errors, winds and uncertainties in the aircraft characteristics. The complexity of the scheme is reduced considerably by neglecting the delays of the controlled aircraft in responding to a velocity command, and by using ground range as the independent variable. The storage and computational requirements for implementing the scheme are quite modest so that data for a number of alternate nominal trajectories could easily be carried on board.

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